TOPOLOGY II FINAL EXAM

Total Marks: 100

Attempt all questions

(1) Describe a CW complex structure on $\mathbb{R}\mathrm{P}^n$, write the associated cellular complex, compute the differential maps, and hence the homology groups of these spaces. Also compute the fundamental group of $\mathbb{R}\mathrm{P}^n$ (for all $n > 0$). (16+4 =20 marks)

(2) Use the Mayer-Vietoris long exact sequence and the excision theorem to compute the homology groups of S^n (for all $n \geq 0$) in two ways. (7+7 =14 marks).

(3) Compute the homology groups and the fundamental group of the g-holed torus M_g (for all $g \geq 0$). (10+4 =14 marks)

(4) Compute the relative homology groups of the pair (X, A) where $X = S^1 \times S^1$ is a torus and $A = S^1 \times \{pt\}$ is a subspace of X. (10 marks)

(5) Does there exist a retraction from the Mobius strip to its boundary circle? Justify your answer. (10 marks)

(6) Compute the homology groups and the fundamental group of the triangular parachute obtained from the standard 2-simplex Δ^2 by identifying its three vertices (10+4 =14 marks)

(7) Find the homology groups of the Klein bottle with coefficients in a field F of characterstic $p > 0$. Compute the fundamental group and describe the universal covering map (you can draw a picture) of the Klein bottle. (10 + 8 = 18 marks)